Byrne Kaulu

Corporate Finance

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Chapter 2
The Time Value of Money

Learning Outcomes

After reading this chapter, you should be able to:

- Explain the concept of the time value of money (TVM) and why it is central to finance.
- Distinguish between present value and future value.
- Apply the formulas for compounding and discounting.
- Understand and calculate annuities and perpetuities.
- Differentiate between ordinary annuities and annuities due.
- Use effective annual rates and understand continuous compounding.
- Apply time value of money concepts to real-life corporate finance decisions.

2.1 Introduction

The time value of money (TVM) is one of the most important principles in corporate finance. It is based on the idea that a kwacha, dollar, or pound today is worth more than the same amount in the future. This occurs because money can earn interest when invested, inflation erodes purchasing power, and risk makes future payments uncertain.

For example, if you receive ZMW 1,000 today and invest it at 10% per year, after one year it will be worth ZMW 1,100. If instead you were promised ZMW 1,000 one year from now, it would not be as valuable today because you lose the opportunity to earn interest on it during the year.

This principle underpins all areas of finance: valuing projects, pricing bonds and stocks, making investment decisions, and structuring loans. Without a solid understanding of TVM, it is impossible to study corporate finance effectively.

2.2 Future Value (FV)

The future value of money represents how much an amount today will grow to after earning interest for a given period. If you invest a sum of money PV (present value) at an interest rate r per period for n periods, the future value is calculated as:

$$FV = PV\,(1 + r)^n \tag{2.1}$$

This process is called compounding, since interest earns additional interest over time.

Example 2.1 Suppose you invest ZMW 1,000 at 12% annual interest for 3 years.

$$FV = ZMW1,000\,(1 + 0.12)^3 = ZMW1,000\,(1.4049) = ZMW1,404.90$$

Thus, the money grows by more than ZMW 400 due to compounding.

2.3 Present Value (PV)

The present value is the reverse process: it tells us how much a future sum is worth today. This is known as discounting. The formula is:

$$PV = \frac{FV}{(1 + r)^n} \tag{2.2}$$

Example 2.2 If you are promised ZMW 2,000 in 4 years and the discount rate is 10% per year, the present value is:

$$PV = \frac{42,000}{(1 + 0.10)^4} = \frac{2000}{1.4641} = ZMW1,366.03$$

This means ZMW 2,000 received in four years is equivalent to only ZMW 1,366 today.

2.4 The Power of Compounding and Discounting

Compounding demonstrates how money grows over time, while discounting shows the erosion of value as payments are delayed. These concepts highlight why financial managers prefer receiving cash sooner and paying obligations later. The longer the time period and the higher the interest rate, the greater the difference between present and future values.

2.5 Ordinary Annuities

An annuity is a series of equal payments made at regular intervals. Examples include loan repayments, rent, and pension payments. There is an ordinary annuity and an annuity due. An ordinary annuity is where payments or receipts (depending on whether it is income or an expense) are made at the end of the period.

2.5.1 Future Value of an Ordinary Annuity

If you receive a fixed payment P each period for n periods at interest rate r, the future value is:

$$FV_{\text{ordinary annuity}} = P\left[\frac{(1+r)^n - 1}{r}\right] \tag{2.3}$$

Example 2.3 You save ZMW 500 each year for 5 years at 8% interest. Calculate how much the savings will be in five years' time.

$$FV_{\text{ordinary annuity}} = 500\left[\frac{(1+0.08)^5 - 1}{0.08}\right] = ZMW2,933.3$$

2.5.2 Present Value of an Ordinary Annuity

The present value of equal payments P received for n years is:

$$PV_{\text{ordinary annuity}} = P\left[\frac{1 - (1+r)^{-n}}{r}\right] \tag{2.4}$$

Example 2.4 You will receive ZMW 1,000 annually for 4 years, with a discount rate of 10%. Calculate the present value of these receipts.

$$PV_{\text{ordinary annuity}} = 1,000\left[\frac{1\ - (1+0.1)^{-4}}{0.1}\right] = \ 1,000\times 3.1699 = ZMW3,169.90$$

2.6 Annuity Due

An annuity due is an annuity where payments are made at the beginning of each period instead of the end. The formulas are the same as above but multiplied by (1+r)

$$PV_{\text{annuity due}} = PV_{\text{ordinary annuity}} \times (1+r) \tag{2.5}$$

$$FV_{\text{annuity due}} = FV_{\text{ordinary annuity}} \times (1+r) \tag{2.6}$$

This adjustment reflects the fact that each payment in an annuity due is invested (or received) one period earlier.

2.7 Perpetuity

A perpetuity P is a stream of equal payments that continues forever. While rare in practice, some financial instruments (such as certain government bonds) approximate perpetuities.

The present value of a perpetuity is:

$$PV_{\text{perpetuity}} = \frac{P}{r} \tag{2.7}$$

Example 2.5 A firm issues preference shares that pay a dividend of ZMW 50 each year, indefinitely. If the required return is 5%, the value of one share is:

$$PV_{\text{perpetuity}} = \frac{50}{0.05} \ = \ ZMW1,000$$

2.8 Effective Annual Rates and Continuous Compounding

When interest is compounded more frequently than annually (for example, semi-annually, quarterly, monthly, or daily), the effective annual rate (EAR) captures the true annual return:

$$\text{EAR} = \left(1 + \frac{r}{m}\right)^{m} - 1 \tag{2.8}$$

where r is the nominal rate and m is the number of compounding periods per year.

Example 2.6 If the nominal rate of 12% is compounded monthly, what is the EAR?

$$\text{EAR} = \left(1 + \frac{0.012}{12}\right)^{12} - 1 = (1.01)^{12} - 1 = 1.1268 - 1 \ \approx 0.1268 \ = \ 12.68$$

For continuous compounding, the formula is:

$$\text{FV} = \text{PV} \times e^{rt} \tag{2.9}$$

where e is the mathematical constant (≈ 2.718).

2.9 Some Applications of Time Value of Money

The time value of money underlies nearly every financial decision. Capital budgeting decisions use discounting to evaluate projects, bond valuation relies on present value of coupon payments, and retirement planning uses annuities. In Zambia, for example, the National Pension Scheme Authority (NAPSA) must apply time value of money concepts when projecting future retirement pay outs and setting contribution rates. Similarly, banks calculate loan repayment schedules using annuity formulas.

2.10 Chapter Summary

The time value of money reflects the idea that money today is worth more than the same amount in the future due to interest, inflation, and risk. The concepts of present value and future value form the basis of compounding and discounting. Annuities, perpetuities, and variations such as annuities due extend these principles to streams of cash flows. Understanding effective interest rates and continuous compounding ensures accurate valuation of investments. Ultimately, mastery of TVM equips financial managers to make informed decisions about investments, financing, and risk management.

Review Questions

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1. Why is a kwacha today worth more than a kwacha tomorrow?
2. Distinguish between present value and future value.
3. Compute the future value of ZMW 10,000 invested at 9% for 7 years.
4. What is discounting, and why is it important in finance?
5. Differentiate between an ordinary annuity and an annuity due.
6. Calculate the present value of receiving ZMW 2,000 per year for 10 years if the discount rate is 6%.
7. What is the present value of a perpetuity paying ZMW 150 annually at a discount rate of 12%?
8. Explain the difference between nominal interest rate and effective annual rate.
9. What is the formula for continuous compounding, and when is it used?
10. Give one practical example in Zambia where time value of money is applied.